Section 13.5

Lines in Space

Example 13.5.1. Find the equation of the line passing through the points $P(x_0, y_0, z_0)$ and $Q(x_1, y_1, z_1)$.

So, precisely what we need in order to determine a line is a point P and a direction $\vec{v}$.

Equation of a Line
A **vector equation** of the line passing through the point P in the direction of $\vec{v}$ is $\vec{r} = \overrightarrow{OP} + t\vec{v}$. That is

$$\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle v_1, v_2, v_3\rangle$$

Consider a point Q and a line l given by $\vec{r} = \overrightarrow{OP} + t\vec{v}$. What is the distance between Q and l?

Planes in $\mathbb{R}^3$

Definition: Plane in $\mathbb{R}^3$
Given a fixed point P_0 and a nonzero **normal vector** $\vec{n}$, the set of points P in $\mathbb{R}^3$ for which $\overrightarrow{P_0P}$ is orthogonal to $\vec{n}$ is called a **plane**.

Example 13.5.2. Find an equation of the plane passing through $P_0(2, 03, 4)$ that is perpendicular to the line $x = 3 + 2t, y = -4t, z = 1 - 6t$.

Example 13.5.3. Let Q be the plane given by $2x - 3y - z = 6$.

1. Find a vector normal to Q.

2. Describe the set of points at which Q intersects the yz-plane.

Example 13.5.4. Find an equation of the plane Q that passes through the point $P(-2, 4, 1)$ and is parallel to the plane $R : 3x - 2y + z = 4$.